Growth Equations: A Quantile Regression Exploration

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Abstract

Estimation of the regression quantiles for the growth equation using broadly constituted samples shows evidence of unconditional income convergence for countries in the upper tail of the conditional growth distribution but not for countries in the lower tail. This finding is in contrast with results obtained from ordinary least squares estimation that show no evidence of convergence. The estimated regression quantiles for the conditional growth equation shows that the effect of control variables on the GDP growth rate varies significantly along the conditional growth distribution. These findings are suggestive of the potential information gains associated with the estimation of the entire conditional growth distribution, as opposed to the conditional mean only.

Keywords: Income Convergence, Quantile Regression, Conditional Distribution of Growth Rates

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1. Introduction

According to the Solow (1956) growth model, economies with similar preference and technological parameters should converge to the same long-run equilibrium income per capita level. This prediction of the neoclassical model became known as the "convergence hypothesis". Since the seminal work of Baumol (1986), testing income convergence has been a recurrent topic in the research agenda of growth economists. The most basic test of income convergence consists of estimating a cross-section regression model where the dependent variable is the average GDP growth rate and the explanatory variable is the level of income. Whenever the slope coefficient is negative, we say that there is evidence of unconditional income convergence. By adding control variables to the regression model, we can test for conditional income convergence: after controlling for countries' differences in steady state, initial income levels should be negatively related with growth rates.

In this article, we estimate the unconditional growth equation for broadly constituted samples using quantile regression. The estimated quantile regression process on the initial income exhibits a concave shape that crosses the zero line at approximately the 70^{th} quantile. This finding suggests that there is evidence of unconditional income convergence for countries in the upper tail of the conditional distribution of growth rates but no evidence of convergence among countries in the lower tail. This result is in contrast with previous estimates obtained with conditional mean estimation methods such as ordinary least squares, henceforth OLS. For instance, Baumol (1986) and Barro (1991) show that there is no evidence of unconditional convergence for broadly constituted samples.

Moreover, estimation of the conditional growth equation yields a quantile regression process on the initial income coefficient also exhibiting a sharp concave pattern. In this case, the quantile regression estimates are all below the zero line, suggesting evidence of conditional convergence for all quantiles. However, the concavity pattern implies that the coefficient on the initial income increases in absolute values with the quantiles, suggesting that convergence is stronger, in some sense, for countries in the upper quantiles.

The motivation to use quantile regression on the growth equations is twofold. First, the quantile regression estimator is robust to outlying observations on the dependent variable. This is an important point given that the unconditional growth distribution is characterized by long right tails, as can be seen, for instance, in Barro and Sala-i-Martin (1995). Second, the quantile regression estimator gives, potentially, one solution to each quantile. Therefore, we may assess how policy variables affect countries according to their position on the conditional growth distribution. This is an interesting way of capturing countries' heterogeneity. There is nothing on the theory of growth saying that the effect of an increase in human capital, for example, should be the same across countries. In fact, we expect it to depend on the specifics of each economy, such as its level of development or its growth rate.

An article related to ours is by Mello and Novo (2001). They employ the inferential procedures recently developed by Koenker and Xiao (2002) to test if the location shift model applies to the convergence growth equation. The location shift model corresponds to the case where the policy variable affects only the mean of the conditional growth distribution. If this is really the case, then estimation of the growth equation using OLS methods is adequate. However, Mello and Novo (2001) find that for commonly used data sets such as the Barro and Lee, and the Summers and Heston, the location shift hypothesis for the growth equation is overwhelmingly rejected. That is, policy variables affect the conditional growth distribution in more complex ways than a shift in the conditional mean. This result provides further motivation to use quantile regression techniques on the growth equation.

In this article, we also revisit the empirical studies of Barro (1991) and Mankiw, Romer and Weil (1992) using quantile regression methods. We find ample evidence that the effect of policy variables on growth rates varies across the quantiles. In some cases, the regression quantile process exhibits a non-linear pattern, while in others it displays a monotonic trend. For example, in section 4, we find that the regression quantile process for a proxy for human capital, primary school enrolment (pri60), exhibits an upward trend. This suggests that the effect of such measure of human capital is stronger for countries in the upper tail of the conditional growth distribution. Estimation under conditional mean methods such as OLS can only capture the effects of policy variables on the mean of the conditional growth distribution but not in any other distributional aspect. That is, traditional conditional mean estimation methods give an incomplete picture of the relationship between the policy variables and growth rates.

This article is divided as follows. In the section 2, we briefly discuss the quantile regression estimation method and some of its properties. In section 3, we present estimates of the regression quantiles for the unconditional growth equation. In section 4 we revisit the “Barro” equations with quantile regression, and in section 5 we do the same for the celebrated Mankiw, Romer and Weil (1992) article. Finally, section 6 concludes.

2. A Brief Introduction to Quantile Regression

In this section we briefly discuss the quantile regression estimation procedure and some properties of the quantile regression estimator[2]. The τ^{th} quantile, for $0 < \tau < 1$, is defined as $Q(\tau) = \inf\{y : F(y) \geq \tau\}$, where Y is a random variable with distribution function given by $F(y) = P(Y \leq y)$. The definition of quantile simply says that an observation in the τ^{th} quantile is greater than $\tau\%$ of the observations and smaller than $(1-\tau)\%$ of the observations.

Consider the linear regression model $y_i = x_i^{'}\beta + u_i$ for i=1,…,n, where β is a Kx1 vector of coefficients, x_i is the column vector that is the transpose of the i[th] row of the X_{nxK} matrix of explanatory variables (or policy variables), y_i is the i[th] observation of the dependent variable, and u_i is the i.i.d. error term. The OLS estimator can be found by choosing the vector β that minimizes the sum of the squares residuals, that is $\min_{\beta \in \Re^K} \sum_{i=1}^{n} (y_i - x_i^{'}\beta)^2$. In contrast, the quantile regression estimator minimizes an asymmetric linear penalty function given by (1) below.

$$\min_{\beta \in \Re^K} \left[\sum_{i \in \{i: y_i \geq x_i\beta\}} \tau \left| y_i - x_i\beta \right| + \sum_{i \in \{i: y_i < x_i\beta\}} (1-\tau) \left| y_i - x_i\beta \right| \right] \qquad (1)$$

The quantile function is a weighed sum of the absolute value of the residuals. It is easy to interpret the quantile function by looking at a particular case of (1). When $\tau = 1/2$, the minimization problem above reduces to $\min_{\beta \in \Re^K} \sum_{i=1}^{n} \left| y_i - x_i^{'}\beta \right|$, which yields

[2] The reader is referred to Koenker and Hallock (2001), and Buchinsky (1998) for recent surveys on quantile regression, and to Koenker and Bassett (1978) for the seminal article.

the conditional median as the solution (that is, the least absolute deviation estimator or median regressor). Note that the absolute value function is a symmetric penalty function. Thus, when the linear penalty function is symmetric we obtain the conditional median as the solution to (1). Following this logic, one would conjecture that the solution to an asymmetric penalty function would yield the other quantiles. It turns out that this conjecture is correct, and it is the intuition behind the quantile function.

By varying the parameter τ on the (0,1) interval we can generate all the regression quantiles and, therefore, obtain the conditional growth distribution of y given x. The coefficient on the k^{th} policy variable, $\beta_k(\tau)$, can be interpreted as the marginal change in the dependent variable due to a marginal change in the k^{th} policy variable conditional on being on τ^{th} quantile. Since potentially we have one β for each τ, the quantile regression approach allows us to identify the effects of the covariates on the dependent variable at different points on the distribution. For instance, suppose that the dependent variable is the average growth rate in income per capita and the explanatory variable is the initial level of income per capita. The coefficient on initial income at $\tau = 0.10$, $\beta_k(0.10)$, gives the marginal change in average GDP growth rate associated with a marginal change in initial income for countries that are in the bottom 10% of the conditional distribution of the average GDP growth rate[3]. In section 3, we present evidence suggesting that in fact the slope coefficients change substantially across the quantiles. This parameter heterogeneity is an interesting way to analyze the effect of policy variables on the growth rate, especially when one is estimating the correlation between growth rates and some key policy variables such as human capital. We can also

[3] From now on we shall call countries on the left tail of the conditional distribution of GDP growth rates slow-growing countries, and countries on the right tail of the distribution fast-growing countries.

infer about the speed of convergence at different points on the conditional growth distribution. Under mean regression methods such as OLS the coefficient on the policy variables is constrained to be the same for all quantiles, and a wealth of information is left untouched.

It is important to understand when estimation of the regression quantiles is superior to estimation of the conditional mean. If the policy variables affect only the location of the conditional growth distribution as in the classical homoskedastic linear regression model then conditional mean estimation methods are appropriate[4]. However, when the policy variables affect the distribution of the error term then conditional mean estimation methods are no longer adequate. Policy variables can affect conditional growth distribution in a number of ways, for instance, they can affect the dispersion, the skewness, stretch one tail, fatten the other, etc. In this case, one would like to estimate the entire conditional growth distribution, so that the use of quantile regression would be more appropriate then conditional mean estimation methods.

As discussed in Koenker and Hallock (2001), an attractive property of the quantile regression estimator is its robustness to the presence of outlying observations on the dependent variable. While OLS estimator magnifies the effect of outliers, the quantile regression estimator penalizes tail observations. This property of the quantile regressor estimator is particularly important in our application given the fact that the distribution of average GDP growth rates is skewed to the right[5].

[4] This is Koenker and Xiao's (2002) location shift model discussed above.

[5] In the Summers and Heston data set version 6.1 the cross-section distribution of the average GDP per worker for the period 1960-1998 has a skewness of 0.3475 indicating a long right tail. For the period 1950-1998, the same variable, exhibits a skewness of 0.44 and a kurtosis of 2.455, indicating fat tails, and slightly skewed to the right.

3. The Unconditional Growth Equation[6]

We use the variable RGDPWOK (Real GDP chain per worker) from the Summers and Heston data set (PWT) mark 6.1[7] to estimate the basic unconditional growth equation. We work with three samples: the first, which we call "large_50", includes 51 countries for which data on RGDPWOK is available for the period 1950-1998; the second sample, called "large_60", includes 104 countries for which the RGDPWOK data is available for the period 1960-1998; and the third sample, which we call "stacks", consists of the pooled average growth rate for four sub-periods: 1960-70, 1970-80, 1980-90, and 1990-98. We also divide the samples in two other groups, one consisting of OECD countries[8], and the other with non-OECD countries. Table 1 displays the OLS estimates of the growth equation for three samples described above.

[Insert table 1]

The results in table 1 confirm the previous OLS estimates in Baumol (1986) and Barro (1991). The initial income coefficient is negative for the large_50 sample, and positive for the large_60 and the stacks samples. However, in all cases it is not statistically different from zero. Thus, when the sample consists of a large cross-section of countries we cannot find evidence of convergence. This result is not really surprising since one cannot hope that all economies in a broadly constituted sample will share

[6] We are well aware of the limitations of regressing growth rates on levels as a way of assessing income convergence, namely, that from a negative slope coefficient one cannot conclude that the cross-sectional dispersion of income is shrinking. See Friedman (1992) and Quah (1993) for more on that. However, in this article we avoid a more elaborate discussion on that to concentrate on the results and interpretations of the application of the quantile regression technique on the growth equations. Although, later in this section, we try to link the shape of the quantile regression process on the initial income coefficient with σ-convergence.

[7] The Summers and Heston data set is available at http://pwt.econ.upenn.edu/Downloads/index.htm.

[8] Our OECD sample includes 25 out of the 30 member countries. We exclude Poland, Hungary, Czech Republic, Slovak Republic, and Turkey, to make our sample consistent with previous studies.

similar tastes and technological parameters -- a necessary pre-condition for income convergence.

For the OECD samples we find that the initial income coefficient is negative and statistically significant, suggesting evidence of convergence. This result is also consistent with previous estimates of the growth equation, for instance, as in MRW. From the initial income coefficient we can obtain the speed of convergence and the half-life[9]. In a linear OLS regression of the average growth rate on the initial income level, the slope coefficient is related to the speed of convergence β according to the following formula: $-(1-e^{-\beta T})/T = b$, where T is the sample period, and b the OLS estimate of the initial income coefficient. For the OECD_50 and OECD_60 the speed of convergence is, respectively, 3% and 3.12%. These estimates give a half-life of approximately 23 and 22 years, respectively. MRW's estimate for the speed of convergence is 2.02% with an implied half-life of 34 years. Thus, according to our estimates the OECD countries are converging at a faster pace for the period 1960-98 than for the period 1960-85.

We also report the OLS estimates of the growth equation for non-OECD samples. The idea of estimating the growth equation for non-OECD samples is to assess the extent that OECD countries can influence estimates of the initial income coefficient in large samples. According to the results in table 1, this influence is not strong. In fact, in non-OECD equations the estimated coefficient on initial income is negative with better t-statistics than the ones from the large sample regressions. We will return to this

[9] The half-life is the number of years that the economy takes to transit half way to its steady state level of income per capita.

point later. Figures 1-3 display the regression quantile processes for the unconditional growth equation for the large_50, large_60, and stacks samples.

[Insert figures 1-3]

Each figure exhibits the entire quantile regression process on the initial income variable, the 95% confidence interval for the quantile regression estimate, and the OLS estimate on the initial income (dashed line). The first important observation is that for all samples the quantile regression processes exhibit the same concavity pattern. It is positive for countries in the lower tail of the conditional growth distribution and negative for countries in the upper tail[10].

For the large_50 sample (figure 1), the estimated coefficient on the initial income for the median ($\tau = 1/2$) of the conditional growth distribution is -0.0003. This coefficient means that a one percent increase in the initial income per capita is associated with a reduction of 0.0003% of the growth rate in income per capita per year. This coefficient increases in absolute value to –0.0074 in the 95th quantile, almost 25 times its value at the 50th quantile. To have an idea of the magnitude of this increase it is useful to calculate the speed of convergence and the half-life associated with each estimate. For countries on the top 50% of the distribution the half-life is approximately 2,310 years. This corresponds to a close to zero speed of convergence[11]. On the other hand, for countries on the top 95% of the distribution the speed of convergence is 0.91% per year, and the associated half-life 76 years.

For the second sample (large_60), the initial income coefficient is negative for the top 25% fast-growing countries but positive for the other quantiles. This finding

[10] Although the OLS estimates lie within the quantile regression confidence intervals, recall again the results in Mello and Novo (2001) showing that Koenker and Xiao's (2002) location shift test corresponding to the OLS model is rejected for this equation.

contrasts with the OLS estimate in table 1, which suggests that there is no evidence of convergence for this set of countries. Here one can see the information gain provided by estimation of the entire conditional distribution of growth rates: when one is constrained to look at the conditional mean only it is left with the impression that there are no convergence forces for broadly constituted samples. This does not seem to be the case for countries in the upper quantiles of the growth distribution. For instance, for countries on the top 95% of the growth distribution the speed of convergence is 1.00% and the associated half-life is 69 years.

The estimates for the third sample ("stacks"), shown in figure 3, largely confirm our previous findings. The motivation to stack the data is to capture growth patterns within the four time intervals, 1960-69, 1970-79, 1980-89, and 1990-98[12]. Visual inspection suggests that the concavity of the regression quantile process is more dramatic in figure 3 than in then previous figures. In this case, the initial income coefficient is negative for countries in the top 40% of the growth distribution. Moreover, the OLS estimate lies outside the quantile regression confidence interval for the top 25% fast-growing countries[13]. In conclusion, figures 1-3 suggest that countries in the upper tail of the conditional distribution of growth rates are the driving forces behind income convergence.

Koenker and Machado (1999) have an interesting interpretation of the concavity pattern exhibited by the regression quantile on initial income. They suggest that if one would interpret this concavity as a downwardly sloping effect, "this would imply a

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[11] More specifically, the speed of convergence associated with the $\beta(0.50)$ estimate is 0.03% per year.

[12] Koenker and Machado (1999) also used pooled data in their estimates of the growth equation.

[13] Note that in figures 1 and 2 the OLS estimate lies within the quantile confidence interval. Given that, one might be tempted to conclude that OLS estimation procedure is adequate then. However, the results in Mello and Novo (2001) show that the data grossly violates the statistical model required to validate conditional mean estimation methods.

stronger sense of convergence in which the scale of the international distribution of per capita income would shrink over time". This interpretation clearly has implicit the notion of σ-convergence, that is, a reduction in the cross-sectional variance of income per capita[14]. In this sense, it would be interesting to look at some measures of dispersion of the cross-sectional distribution of income.

[Insert Table 2]

Table 2 above shows the mean, standard deviation, coefficient of variation, and the standard deviation of the log of the variable RGDPWOK from the PWT data set. The coefficient of variation and the standard deviation of the log of income per worker are standard measures of inequality. Their formulas are given by, respectively, $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i-\bar{y}}{\bar{y}}\right)^2}$, and $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\log(y_i/\tilde{y})\right)^2}$, where y_i is the level of GDP per worker, $\tilde{y}$ is the cross-sectional average of the log of GDP per worker, and n is the sample size.

The estimates in table 2 show that the coefficient of variation is relatively stable for the large sample and the non-OECD sample for the period 1960-90, and has an upward trend in the period 1990-1998. For the large sample, it decreases from 0.93 in 1960 to 0.86 in 1980, to increase again to 0.97 in 1998. For the non-OECD sample the coefficient of variation decreases from 0.83 in 1960 to 0.81 in 1990, to increase again in 1998 to 0.93. For OECD sample we have a different picture. There is a sharp decline in the coefficient of variation from 1960 to 1990, from 0.40 to 0.22, respectively. However, it increases again to 0.28 in 1998 to 0.12.

[14] It is well known in the literature the relationship between β-convergence and σ-convergence - the former is necessary but not sufficient for the latter. The convergence concept embedded in the OLS growth equation is the β-convergence.

The standard deviation of the log for the large sample and the non-OECD sample, except for the period 1980-90 for the non-OECD sample, increases monotonically over the entire period. The coefficient of variation exhibits a distinct pattern; at certain points goes in the opposite direction of the standard deviation of the log. It decreases first, reaching its lowest level in 1980, and then it increases to its highest level in 1998, for both samples. Dalgaard and Vastrup (2001) show that these two measures of dispersion attach different weights to intra-distribution dynamics, and because of that can exhibit a contradictory behavior. More specifically, Dalgaard and Vastrup (2001) show that for the period 1960-91 poor countries became poorer and rich countries became richer, and that there has been considerable catching up between the middle and the top of the world income distribution. The different weights attached by the coefficient of variation and the standard deviation of the log to these intra-distribution dynamics explain the divergent path of these two measures for the period 1960-80.

On the other hand, for the OECD sample the standard deviation of the log and the coefficient of variation exhibit a similar pattern. The standard deviation of the log decreases from 0.22 in 1960 to 0.10 in 1990, to increase again in 1998 to 0.12. The estimates in table 2 suggest that the cross-sectional distribution of GDP per worker for the OECD countries has shrunk between 1960 and 1998, while the cross-sectional distribution of GDP per worker for the large sample and the non-OECD sample grew more disperse over time. Thus, based on the estimates in table 2, we can conclude that σ-convergence is taking place among OECD countries but the same is not true for the rest of the world.

One interesting exercise is to decompose the OLS estimator of the slope coefficient in the unconditional growth equation in order to see how much each country contributes to total estimate. The OLS estimator can be written as: $\sum_{i=1}^{n}\left[\frac{(y_i-\bar{y})}{(x_i-\bar{x})}\right]\frac{(x_i-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$, where y_i is the average of the log difference in income per capita between the initial and the final period for country *i*, $\bar{y}$ is the cross-sectional average for the *y* variable, x_i is the log of the initial income per capita for country *i* as defined in table 1, $\bar{x}$ is the cross-sectional average for the variable *x*, and n is the sample size[15]. Because the second term in the expression above represents the weight of country *i* on the OLS estimator, a converging economy is one where the first term is negative. That is, converging economies have their initial incomes above the cross-sectional average and their growth rates below the cross-sectional average, or their initial income below the cross-sectional average and their growth rates above the cross-sectional average. In the large_60 sample, there are 45 converging economies according to this criterion, corresponding to 43% of the sample size. Only 8 out of the 24 OECD members are in the converging economies group. Moreover, the OECD countries represent only 13% of the total contribution of all the converging economies to the magnitude of the estimated β.

However, defining convergence according to the formula above is problematic because it makes the definition of convergence sensitive to the sample selection. For instance, when the sample consists of OECD countries only, we find that 18 out of the 24 economies are converging according the criterion above. This is in sharp contrast

[15] This derivation follows Bernard and Durlauf (1996).

with the number of OECD converging economies found in the large sample (8 out of 24, as mentioned above). This discrepancy is no statistical puzzle. It happens because each sample has a different mean growth rate and average initial income, which ultimately determines whether the economy is converging or not. This remark stresses how evidence of convergence based on a regression of growth rates on levels depends explicitly on the sample selection. This point was made by Hotelling (1933) 70 years ago, and cited more recently by Friedman (1992). Interestingly, they suggest looking directly at measures of dispersion such as the coefficient of variation as a real test of convergence "The real test of a tendency to convergence would be in showing a consistent diminution of variance,…", p. 464, Hotelling (1933).

4. The "Barro" equations with Quantile Regression

In this section, we revisit the growth equations in Barro (1991) using quantile regression. We use the variable RGDPSH5 (real GDP per capita) for the period 1960-1985[16] from the Barro and Lee data set[17], henceforth BL. Table 3 displays the OLS estimates of the growth equation. Equation 1 is the unconditional growth equation with the average growth rate of GDP per capita as the dependent variable and initial GDP per capita as the only explanatory variable. Equations 2-6 add control variables to the basic equation. The variables sec60 and pri60 are proxies for human capital denoting, respectively, the 1960 values of school-enrolment rates at the secondary and the primary

[16] We chose the same period analyzed by Barro (1991) to make our estimate comparable to his. However, the data set we used is an update of the original data in Barro (1991). Therefore, one should expect some small discrepancies between our estimates and his, but the results should be qualitatively similar.

[17] The BL data set is available at http://www.nber.org/pub/barro.lee/ZIP/.

levels[18]. The variable gcy is the ratio of real government consumption expenditure to real GDP. The variables rev and assass are proxies for political instability. The former is the number of revolutions and coups per year, and the latter is the number of political assassinations per year per millions of habitants. The variable ppidev60 is a measure of market distortion based on the deviations of the investment price deflator from its sample mean. The variable inv is the average share of investment to real GDP for the period 1960-85. The variable pop is the average growth rate of the total population for the period 1960-85. The reader is referred to Barro (1991) for more details. Our sample contains observations on 100 countries. It is the same across equations, and was determined according to data availability.

[Insert table 3]

The estimates in table 3 are consistent with the ones in Barro (1991). For the large sample (equation 1), the estimated coefficient on the initial income is positive and not significant, suggesting lack of income convergence. Adding all the control variables, except inv and pop (equation 2), the coefficient on the initial income becomes negative and highly significant suggesting evidence of conditional convergence. Moreover, in equation 2, all the estimated coefficients on the control variables have the expected sign and are consistent with Barro's estimates. For instance, Barro's estimate of the sec60 coefficient in equation 2 is 0.0305 with a t-statistic of 3.86 while ours is 0.0327 with a t-statistic of 2.73. For the pri60 variable, his estimated coefficient is 0.0250 with a t-statistic of 4.46 while ours is 0.0217 with a t-statistic 2.86. His estimate of the initial income coefficient is -0.0075 with a t- statistic of –6.25, while ours is –0.0047 with a t-

[18] These variables are computed as the ratio of the number of students enrolled in the designated grade levels over the total of the population of the corresponding age group.

statistic of -4.7. Analysis of variance (ANOVA) tests for the control variables in equation 2 reject the null hypothesis that all controls are jointly not significant.

The remaining equations are used to test for the validity of the control variables listed above. Equation 6 includes all the control variables, and is used to test for the joint significance of the controls rev, assass, ppidev60, and gcy (equation 5 is the restricted model). The ANOVA test statistic indicates that at least one of these control variables is significant. In further tests not shown in table 3, we find that the variables rev, assass, and ppidev60 are not statistically significant, suggesting that the variable gcy is driving the result in the ANOVA test above. Equation 4 is the same as equation 6 except that it excludes the variable pri60. We compare it with equation 3, and reject the null hypothesis that the coefficients on rev, assass, ppidev60, and gcy, are jointly insignificant. Similarly as above, in further tests (not shown in table 3), we find that the variables rev, assass and ppidev60 are jointly statistically insignificant, and that the only control variable that is significant is gcy. We now turn to the quantile estimates of the equations in table 3.

[Insert figure 4]

Figure 4 exhibits the quantile regression process on the initial income coefficient for the unconditional growth equation in table 3 (equation 1). It has the same concavity pattern as before. Note that the OLS estimate of 0.0008, lie outside the quantile regression confidence interval for the 40% fast-growing countries. However, the quantile regression estimates suggest that for the top 30% fast-growing countries the initial income coefficient is –0.0013, for the top 20% fast-growing countries the coefficient is –0.0011, and for the top 10% fast-growing countries is –0.0026. Since the initial income variable is measured in thousands of constant dollars, this coefficient means that for the top 10% fast-growing countries a $1,000 increase in initial income is associated with a reduction of 0.26 percentage points in their GDP growth rate per year.

Figure 5 displays the quantile regression processes for the policy variables on second equation in table 3. The quantile regression process on the initial income lies below the zero line for all quantiles, suggesting evidence of conditional convergence. Moreover, it exhibits the same concavity pattern as before, although not as pronounced as in the unconditional equation. This suggests that convergence is stronger, in some sense, for countries in the upper tail of the conditional growth distribution.

The quantile regression process for the primary school enrolment variable (pri60) has an upward trend suggesting that the association between this measure of human capital and average growth rate is stronger for countries in the upper quantiles, whereas the quantile process for the secondary school enrollment (sec60) is relatively stable around the OLS estimate. This is also true for the proxies for political instability (rev and assass) and market distortions (ppidev60). The effect of government consumption (gcy) is negative for all the quantiles, however it appears to dampen its negative effects for countries above the conditional median.

Figure 6 exhibits the quantile regression processes for the policy variables in equation 3 in table 3. The initial income coefficient is negative for all quantiles, suggesting evidence of conditional convergence for the entire distribution. The proxy for human capital (sec60) is stable across the quantiles. The quantile regression process for the variable inv has an interesting pattern, it is positive for all the quantiles, as expected, however it decreases from τ=30% to τ=70%, and then it increases again. Its peak is reached at the bottom 30% of the conditional growth distribution. Finally, the quantile regression for the coefficient on population growth (pop) increases non-linearly across the quantiles, but never goes above the zero line. Table 4 below displays the ANOVA tests for selected regression quantile estimates for Barro conditional equations.

[Insert table 4]

The ANOVA tests in table 4 are based on the procedures developed in Koenker and Bassett (1982a), and have similar interpretation to the traditional ANOVA tests. Equation 7 is used to test the conditional equation versus the unconditional equation (not shown in table 4). The F-test suggests that the control variables are jointly highly significant for the three quartiles considered. Equations 9 and 11 are used to test the significance of gcy, rev, assass, and ppidev60. We compare equation 9 with equation 8, and equation 11 with equation 10. The variable pri60 is included in equation 11 but not in equation 9. We do this to see if pri60 has any influence on the joint significance of the controls gcy, rev, assass, and ppidev60. We find this set of controls to be highly significant in both cases. However, further tests, not reported in table 4, we suggest that gcy is driving the test results for the first and second quartile. That is, the controls rev, assass, and ppidev60, are jointly insignificant at the first and second quartile, and are jointly significant only at the third quartile. Finally, tests on the slope stability on the quantile regression based on Koenker and Bassett (1982b) indicate that the slope coefficients vary significantly across the quartiles.

5. Mankiw-Romer-Weil equations with Quantile Regression

We use MRW's original data, and an update of their data set provided by Bernanke-Gurkaynak (2001)[19], henceforth BG, to revisit the growth equations in MRW. Table 5 displays our estimates of the MRW equations for the large sample and the

OECD sample. The estimation period is 1960-85 for MRW, and 1960-95 for the BG. Our regression model uses as the dependent variable the average growth rate while MRW uses the total growth rate as the dependent variable[20].

[Insert table 5]

The results in table 5 are consistent with previous estimates. For instance, the estimated coefficient on the initial income in equation 12.A is close to zero and insignificant – that is, for broadly constituted samples one cannot find evidence of unconditional income convergence with OLS estimation. For equation 13.A, we estimate the growth equation with the MRW control variables. The variable inv is the average share of real investment in real GDP. The proxy for human capital (school) is measured as the working-age population (defined as people between the ages of 15 and 64) that is enrolled in secondary school. The variable ngdelta includes the average rate of growth of the working-age population plus 5% to account for the depreciation rate of physical capital and the rate of labor augmenting technological progress. The estimated coefficient on the initial income in equation 13.A is –0.0119, which when multiplied by 25 equals –0.2975. MRW estimate for the same variable is equal –0.289. In equation 14.A all our coefficient estimates are identical to MRW's.

The results using the BG data set are similar to the ones in MRW. For instance, in equation 13.B, the initial income coefficient is equal to -0.0122, which is close to –0.0119 in equation 13.A. Similarly, the initial income coefficient in equation 15.B is -0.0157 while in equation 15.A is –0.0159. We also estimate regression quantiles for the

[19] BG's data set is available at http://www.princeton.edu/~bernanke/data.htm.

[20] Thus, in order to compare our estimated coefficients with theirs we have to multiply our estimated coefficients by 25 (that is, the number of years in MRW's sample period).

equations in table 5, however, to economize on space, we only display in figure 7 below, the quantile regression process for equation 13.B[21].

[Insert figure 7]

The quantile process for the initial income coefficient exhibits an already familiar concave shape. It lies below the zero line for all quantiles, suggesting that there is evidence of conditional convergence at all points on the conditional growth distribution. Interestingly, its concavity suggests that convergence is stronger, in some sense, for countries in the upper tail. The quantile process for the investment share is relatively stable around its OLS estimate. The magnitude of its effect on the tails of the conditional growth distribution are similar to the magnitude its effects on the median. One could interpret this finding as suggesting that, although important to explain mean growth rates, physical capital accumulation is not the key policy variable behind growth miracles. The quantile process for the population growth variable lies below the zero line, and exhibits a slightly upward trend. This pattern suggests that the negative effects of population growth on GDP growth rates tend to dampen for countries in the upper tail of the conditional growth distribution.

The quantile process for the variable school exhibits a non-linear increasing trend. For countries in the bottom 10% of the conditional growth distribution the estimated coefficient on school is 0.0066, it increases to 0.0121 for countries in the conditional median, decreases to 0.0085 for countries in the top 30% of the distribution, to increase again to 0.0133 in the top 10% of the distribution. This result suggests that the effect of human capital, measured by secondary school enrolment, has a stronger impact on countries in the upper tail of the conditional growth distribution.

[21] These figures are available upon request. The quantile processes are similar to the ones in figure 7.

One interesting exercise is look at the speed of convergence and the associated half-life implied by the initial income estimates. Table 6 displays the speed of convergence and the half-life associated with the unconditional growth equation for the OECD sample, and the conditional growth equation for the large sample, using both MRW and BG data sets. The estimates suggest that the speed of convergence has increased from the 1960-1985 to the 1960-1995 period. For instance, the speed of convergence implied in the OLS estimate for the OECD sample increases from 1.67% to 1.81%, and for the large sample it increases from 1.41% to 1.60% (columns 1 and 3, respectively). The quantile process spanning the interval $\tau \in [.20, .80]$ shows that the speed of convergence is globally faster in the BG data set. A similar story holds for the conditional equation for the large sample. An important aspect of the estimates in table 6 is the large difference in the speed of convergence and the half-life observed across the quantiles. For instance, for the large sample, using the BG data set (last column in table 6), the half-life for a country on the bottom 20% of the conditional growth distribution is 63.44 years, while for a country on the top 20% the half-life is 30.89 years. For OECD economies (same column), a country on the bottom 20% of the distribution has a half-life of 35.07 year, while a country on the top 20% has a half-life of 18.31 years.

[Insert Table 6]

ANOVA tests for the MRW equations are shown in table 7. The tests suggest that the proxy for human capital is highly significant. When equation 18 is compared with equation 17, the F-test statistic suggests that, for quartiles and the OLS specification, the augmented equation with human capital is a better specification. When equation 17 is tested against equation 16, we find that investment (sk6095) and population (glf6095) growth are highly significant. Finally, the slope stability tests for the quantiles suggest that for the first two models there is no significant variation of the slope coefficient for the quartiles. However, for the augmented model we reject the null hypothesis of constant slope coefficients for traditional levels of significance.

[Insert Table 7]

6. Conclusion

This article explores the convergence growth equations using quantile regression methods. The motivation to estimate the growth equations using quantile regression is twofold. First, while the OLS estimator magnifies the effects of outlying observations on the dependent variable, the quantile regression estimator penalizes tail observations. The robustness of the quantile regression estimator is particularly important to our application, especially given that the unconditional distribution of growth rates is characterized by long right tails. Second, the quantile regression estimator gives, potentially, a family of quantile coefficients; one for each sample quantile. Each slope coefficient can be interpreted as a different response of the GDP growth rate to a change in a policy variable, according to a country's position on the conditional growth distribution. This is an interesting way of capturing parameter heterogeneity. After all, there is nothing on the theory of growth that says that policy variable should have the same effect across countries.

One particularly interesting result we find is the concavity pattern of the regression quantile process on the initial income coefficient, for both the conditional and the unconditional growth equations. This finding, interpreted in the light of the definition of β-convergence, establishes, in the unconditional growth equation, evidence of convergence for countries in the upper quantiles but not for countries in the lower quantiles. And, for the conditional growth equation where the entire regression quantile process is below the zero line, it establishes evidence of convergence for all quantiles, and that convergence is stronger, in some sense, for countries in the upper quantiles.

This result helps reconcile previous OLS estimates of the growth equation, in particular, Baumol's (1986) "convergence clubs". Consider the unconditional growth equation. In preliminary estimates, we observe that most OECD countries are in the upper tail of the conditional growth distribution. That is, the OECD countries are in the upper quantiles – this is exactly where the quantile regression estimates are negative and the evidence of convergence in OLS equations is strong. When the sample includes only OECD countries, the OLS estimator is significantly negative capturing the strong convergence forces among countries in the upper tail of the conditional growth distribution. That is why the OLS estimate of the initial income coefficient in the unconditional growth equation for OECD samples is significantly negative. When the sample includes all countries in the world the OLS estimate of the initial income coefficient captures the overall no convergence tendency among all countries in the world. Note that, the above remarks are in line with Koenker and Machado's (1999) interpretation of the concavity pattern of the regression quantile process on the initial income.

Finally, our results can be subject to further investigation, and extended in several ways. Application of recent inferential methods in quantile regression such as Koenker and Xiao's (2002) location shift and location-scale shift tests is a natural extension of our work. Moreover, the newest version of the Summers and Heston data set (version 6.1) contains a number of important macroeconomic variables that we did not discuss here. Investigation on how these policy variables relate to GDP per worker growth rates would also be an interesting extension.

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Table 1

Table 1: OLS estimates of the growth equation
Dependent variable: Average log difference in GDP per worker

Sample	**Coefficient on the initial income**	**Sample size**	**Adjusted R^2**
Large_50	-0.00026 (0.0019)	51	-0.0200
Large_60	0.00108 (0.0016)	104	-0.0052
Stacks	0.00052 (0.0011)	416	-0.0019
Non-OECD_50	-0.0031 (0.0024)	30	0.0196
Non-OECD_60	-0.0009 (0.0023)	80	-0.0107
Non-OECD Stacks	-0.0009 (0.0016)	320	-0.0022
OECD_50	-0.0159 (0.0022)	21	0.7231
OECD_60	-0.0184 (0.0025)	24	0.6927
OECD_Stacks	-0.0287 (0.0027)	96	0.5506

Notes: The table reports estimates of the slope coefficient of the following equation $(1/T)\ln(y_{T,i}/y_{0,i}) = \alpha + \beta \ln(y_0) + \varepsilon_i$, where $y_{T,i}$ and $y_{0,i}$ are, respectively, the final period and the initial period GDP per worker at constant international dollars from the PWT data set version 6.1, T is the sample size, and ε_i error term. The standard error appears in parenthesis.

Table 2

Table 2: Measures of Dispersion of the GDP per Worker

Large Sample (n=104)	**1960**	**1970**	**1980**	**1990**	**1998**
Mean	8.63	12.18	14.83	16.58	19.11
Standard deviation	8.06	11.16	12.81	15.31	18.52
Coefficient of Variation	0.93	0.92	0.86	0.92	0.97
Standard deviation of the log	0.43	0.46	0.47	0.48	0.52

Non-OECD Sample (n=80)	**1960**	**1970**	**1980**	**1990**	**1998**
Mean	5.37	7.41	9.14	9.45	10.92
Standard deviation	4.47	6.44	7.34	7.67	10.11
Coefficient of Variation	0.83	0.87	0.80	0.81	0.93
Standard deviation of the log	0.35	0.38	0.40	0.40	0.44

OECD Sample (n=24)	**1960**	**1970**	**1980**	**1990**	**1998**
Mean	19.50	28.11	33.80	40.33	46.41
Standard deviation	7.83	8.44	7.87	9.07	13.17
Coefficient of Variation	0.40	0.30	0.23	0.22	0.28
Standard deviation of the log	0.22	0.16	0.12	0.10	0.12

Notes: The mean is in thousands of constant international dollars. The coefficient of variation is calculated according to the following $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i - \bar{y}}{\bar{y}}\right)^2}$, where y_i is the GDP per worker for country i, and $\bar{y}$ is the cross-sectional average.

Table 3

Table 3: Previous results on convergence: the "Barro" equation
Dependent variable: Average growth rate in real GDP per capita for 1960-1985

Variable	Equation 1	Equation 2	Equation 3	Equation 4	Equation 5	Equation 6
Constant	0.0203 (0.0026)	0.0225 (0.0073)	0.0061 (0.0076)	0.0255 (0.0089)	-0.0007 (0.0083)	0.0193 (0.0100)
y_0	0.0008 (0.0008)	-0.0047 (0.0010)	-0.0038 (0.0010)	-0.0049 (0.0009)	-0.0039 (0.0010)	-0.0049 (0.0009)
sec60	--	0.0327 (0.0120)	0.0210 (0.0131)	0.0189 (0.0123)	0.0152 (0.0132)	0.0156 (0.0125)
pri60	--	0.0217 (0.0076)	--	--	0.0146 (0.0075)	0.0101 (0.0077)
gcy	--	-0.0756 (0.0267)	--	-0.0828 (0.0235)	--	-0.0713 (0.0250)
rev	--	-0.0102 (0.0068)	--	-0.0046 (0.0065)	--	-0.0046 (0.0065)
assass	--	-0.0437 (0.0196)	--	-0.0358 (0.0182)	--	-0.0397 (0.0184)
ppi60dev	--	-0.0062 (0.0039)	--	-0.0027 (0.0037)	--	-0.0031 (0.0037)
inv	--	--	0.0013 (0.0002)	0.0010 (0.0002)	0.0011 (0.0002)	0.0009 (0.0002)
pop	--	--	-0.0015 (0.0022)	-0.0022 (0.0021)	-0.0011 (0.0022)	-0.0019 (0.0021)
ANOVA	--	10.883 (0.000)	--	4.323 (0.003)	--	3.717 (0.008)
Adjusted R^2	0.0003	0.3772	0.3804	0.4565	0.3984	0.4608

Notes: Our sample contains observations on 100 countries while Barro (1991) uses observations on 98 countries. The estimated equation is given by $(\Delta y / y)_i = \alpha + \beta y_{60,i} + \gamma' X_i + \varepsilon_i$, where $(\Delta y / y)_i$ is the average growth rate for the 1960-85 period, y_{60} is the level of income per capita for 1960, and X is the matrix of control variables listed above. Standard errors are in parenthesis.

Table 4: Previous results on convergence: the "Barro" equation
Dependent variable: Average growth rate in real GDP per capita for 1960-1985

Variable	tau	Equation 7	Equation 8	Equation 9	Equation 10	Equation 11
Const.	0.25	0.0225 (0.0072)	-0.0045 (0.0052)	0.0202 (0.0104)	-0.0039 (0.0080)	0.0140 (0.0100)
	0.50	0.0112 (0.0084)	0.0035 (0.0070)	0.0262 (0.0086)	-0.0008 (0.0066)	0.0118 (0.0092)
	0.75	0.0195 (0.0072)	0.0151 (0.0117)	0.0453 (0.0087)	0.0026 (0.0131)	0.0379 (0.0141)
gdp60	0.25	-0.0035 (0.0011)	-0.0034 (0.0006)	-0.0043 (0.0007)	-0.0035 (0.0010)	-0.0042 (0.0006)
	0.50	-0.0037 (0.0009)	-0.0042 (0.0005)	-0.0049 (0.0010)	-0.0037 (0.0004)	-0.0048 (0.0008)
	0.75	-0.0053 (0.0010)	-0.0043 (0.0012)	-0.0061 (0.0010)	-0.0044 (0.0011)	-0.0058 (0.0010)
Sec60	0.25	0.0179 (0.0118)	0.0204 (0.0057)	0.0111 (0.0078)	0.0187 (0.0110)	0.0032 (0.0103)
	0.50	0.0185 (0.0113)	0.0266 (0.0071)	0.0207 (0.0138)	0.0074 (0.0064)	0.0166 (0.0126)
	0.75	0.0336 (0.0107)	0.0181 (0.0155)	0.0291 (0.0128)	0.0249 (0.0152)	0.0254 (0.0137)
pri60	0.25	0.0187 (0.0070)	--	--	0.0041 (0.0083)	0.0125 (0.0082)
	0.50	0.0330 (0.0088)	--	--	0.0160 (0.0068)	0.0114 (0.0071)
	0.75	0.0306 (0.0086)	--	--	0.0252 (0.0126)	0.0153 (0.0109)
Gcy	0.25	-0.1277 (0.0201)	--	-0.0999 (0.0337)	--	-0.0778 (0.0312)
	0.50	-0.0403 (0.0401)	--	-0.0807 (0.0315)	--	-0.0619 (0.0260)
	0.75	-0.0442 (0.0246)	--	-0.0790 (0.0399)	--	-0.0793 (0.0348)
Rev	0.25	-0.0173 (0.0087)	--	-0.0024 (0.0056)	--	-0.0058 (0.0076)
	0.50	-0.0112 (0.0095)	--	-0.0063 (0.0059)	--	-0.0063 (0.0058)
	0.75	-0.0109 (0.0093)	--	-0.0138 (0.0072)	--	-0.0093 (0.0112)
Assass	0.25	-0.0553 (0.0921)	--	-0.0791 (0.0556)	--	-0.0974 (0.0656)
	0.50	-0.0371 (0.0448)		-0.0135 (0.0473)	--	-0.0065 (0.0561)
	0.75	-0.0215 (0.0161)	--	-0.0273 (0.0214)	--	-0.0281 (0.0094)
pppi60dev	0.25	-0.0046 (0.0096)	--	-0.0009 (0.0086)	--	-0.0068 (0.0079)
	0.50	-0.0043 (0.0073)	--	-0.0043 (0.0051)	--	-0.0038 (0.0073)
	0.75	-0.0000 (0.0037)	--	-0.0011 (0.0095)	--	0.0013 (0.0072)
Inv	0.25	--	0.0014 (0.0001)	0.0012 (0.0003)	0.0013 (0.0003)	0.0011 (0.0002)
	0.50	--	0.0013 (0.0002)	0.0010 (0.0002)	0.0011 (0.0002)	0.0011 (0.0002)
	0.75	--	0.0012 (0.0004)	0.0006 (0.0003)	0.0006 (0.0004)	0.0003 (0.0003)
Pop	0.25	--	-0.0020 (0.0016)	-0.0039 (0.0011)	-0.0026 (0.0021)	-0.0040 (0.0017)
	0.50	--	-0.0016 (0.0017)	-0.0024 (0.0017)	-0.0017 (0.0017)	-0.0013 (0.0014)
	0.75	--	-0.0006 (0.0034)	-0.0036 (0.0023)	0.0005 (0.0034)	-0.0035 (0.0028)
ANOVA	0.25	47.675 (0.000)	--	2.540 (0.045)	--	2.236 (0.072)
	0.50	5.938 (0.000)	--	2.048 (0.094)	--	1.793 (0.137)
	0.75	14.852 (0.000)	--	3.493 (0.011)	--	4.554 (0.002)
Slope Stability		1.492 (0.113)	1.186 (0.307)	1.220 (0.252)	1.322 (0.218)	1.402 (0.129)

Notes: The sample size is 100.

Table 5

Table 5: Previous results on convergence: the MRW equation
Dependent variable: Average log difference in GDP per worker

Variable	**Equation 12.A Large Sample**	**Equation 13.A Large Sample**	**Equation 14.A OECD Sample**	**Equation 15.A OECD Sample**
		A. MRW Data Set		
y_0	0.0000 (0.0019)	-0.0119 (0.0020)	-0.0136 (0.0031)	-0.0159 (0.0028)
Inv	--	0.0221 (0.0035)	--	0.0133 (0.0069)
Ngdelta	--	-0.0203 (0.0101)	--	-0.034537 (0.0135)
School	--	0.0087 (0.0024)	--	0.0091 (0.0058)
Adjusted R^2	-0.0097	0.4964	0.4597	0.6512
Sample size	104	104	22	22

Variable	**Equation 12.B Large Sample**	**Equation 13.B Large Sample**	**Equation 14.B OECD Sample**	**Equation 15.B OECD Sample**
		B. BG Data Set		
y_0	0.0045 (0.0022)	-0.0122 (0.0024)	-0.0134 (0.0030)	-0.0157 (0.0025)
Inv	--	0.0143 (0.0025)	--	0.0087 (0.0057)
ngdelta	--	-0.0319 (0.0105)	--	-0.0293 (0.0108)
school	--	0.0098 (0.0024)	--	0.0122 (0.0067)
Adjusted R^2	0.03358	0.5609	0.4923	0.746
Sample size	90	90	20	20

Note: Our dependent variable is the average log difference in GDP per worker, while MRW uses the log difference as dependent variable. In order to compare our estimated coefficients with those in MRW, our estimated coefficient have to be multiplied by 25 (that is, the number of years in MRW's sample period).

Table 6

Table 6: Comparison between the MRW and BG: speed of convergence and half-lives – OECD and large sample

Quantile	MRW Speed of Convergence	MRW Half-life (in years)	BG Speed of Convergence	BG Half-life (in years)
q10	0.69%	100.84	0.43%	161.94
	0.69%	99.75	0.79%	87.68
q20	1.36%	51.09	1.98%	35.07
	1.09%	63.73	1.09%	63.44
q30	1.53%	45.43	2.41%	28.81
	1.19%	58.48	1.63%	42.50
q40	2.06%	33.70	2.31%	30.00
	1.25%	55.36	1.44%	48.02
q50	1.83%	37.88	2.34%	29.62
	1.46%	47.58	1.62%	42.75
q60	1.71%	40.64	2.28%	30.46
	1.56%	44.35	2.25%	30.79
q70	1.90%	36.48	2.30%	30.10
	2.03%	34.15	1.91%	36.30
q80	1.91%	36.35	2.34%	29.67
	2.11%	32.89	2.24%	30.89
q90	5.59%	12.40	3.78%	18.31
	2.22%	31.26	2.95%	23.49
OLS	1.67%	41.54	1.81%	38.26
estimate	1.41%	49.11	1.60%	43.43

Notes: On the first and third columns the observation on the top is the speed of convergence for the unconditional growth equation for the OECD sample, and the observation on the bottom is the speed of convergence for the conditional growth equation for the large sample. On the second and fourth columns the observation on the top is the half-life for the unconditional growth equation for the OECD sample, and the observation on the bottom is the half-life for the conditional growth equation for the large sample. The speed of convergence is calculated according to $-(1-e^{-\beta T})/T = b$, where b is the estimated coefficient, T is the sample period, and β is the speed of convergence. The half-life is calculated according to the formula $-\ln(0.5)/\beta$.

Table 7

Table 7: ANOVA tests for the MRW equations with the BG data

Variable	**tau**	**Equation 16**	**Equation 17**	**Equation 18**
Constant	0.25	-0.0449 (0.0336)	0.1302 (0.0390)	0.1300 (0.0330)
	0.50	-0.0217 (0.0196)	0.0816 (0.0360)	0.1471 (0.0312)
	0.75	-0.0019 (0.0266)	0.0761 (0.0425)	0.1332 (0.0348)
	OLS	-0.0227 (0.0186)	0.0726 (0.0341)	0.1282 (0.0339)
Y60	0.25	0.0060 (0.0038)	-0.0093 (0.0027)	-0.0109 (0.0023)
	0.50	0.0043 (0.0023)	-0.0082 (0.0024)	-0.0131 (0.0022)
	0.75	0.0026 (0.0029)	-0.0100 (0.0029)	-0.0152 (0.0024)
	OLS	0.0045 (0.0022)	-0.0071 (0.0022)	-0.0123 (0.0024)
Sk6095	0.25	--	0.0195 (0.0033)	0.0142 (0.0017)
	0.50	--	0.0203 (0.0020)	0.0118 (0.0031)
	0.75	--	0.0220 (0.0024)	0.0151 (0.0031)
	OLS	--	0.0198 (0.0023)	0.0145 (0.0026)
glf6095	0.25	--	-0.0470 (0.0114)	-0.0402 (0.0100)
	0.50	--	-0.0248 (0.0109)	-0.0368 (0.0087)
	0.75	--	-0.0137 (0.0126)	-0.0179 (0.0101)
	OLS	--	-0.0243 (0.0112)	-0.0316 (0.0103)
Sh6095	0.25	--	--	0.0089 (0.0024)
	0.50	--	--	0.0124 (0.0032)
	0.75	--	--	0.0091 (0.0030)
	OLS	--	--	0.0097 (0.0024)
ANOVA	0.25	--	31.361 (0.000)	14.416 (0.000)
	0.50	--	56.099 (0.000)	15.172 (0.000)
	0.75	--	44.738 (0.000)	9.053 (0.003)
	OLS	--	39.222 (0.000)	49.238 (0.000)
Slope Stability		0.376 (0.687)	1.733 (0.114)	3.975 (0.000)

Notes: The sample size is 90 for all equations.

Figure 1 – Large_50

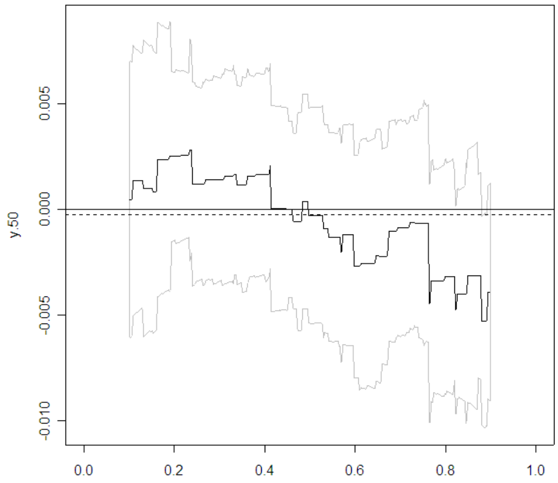

Figure 2 – Large_60

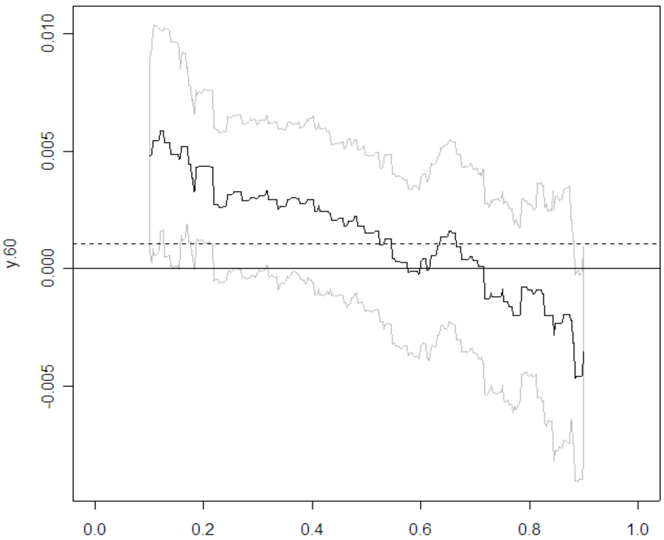

Figure 3 - Stacks

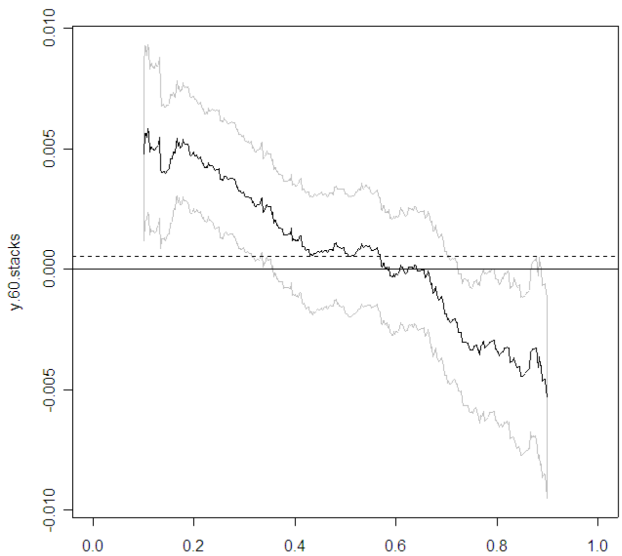

Figure 4 – Table 3, Equation 1
Barro Unconditional

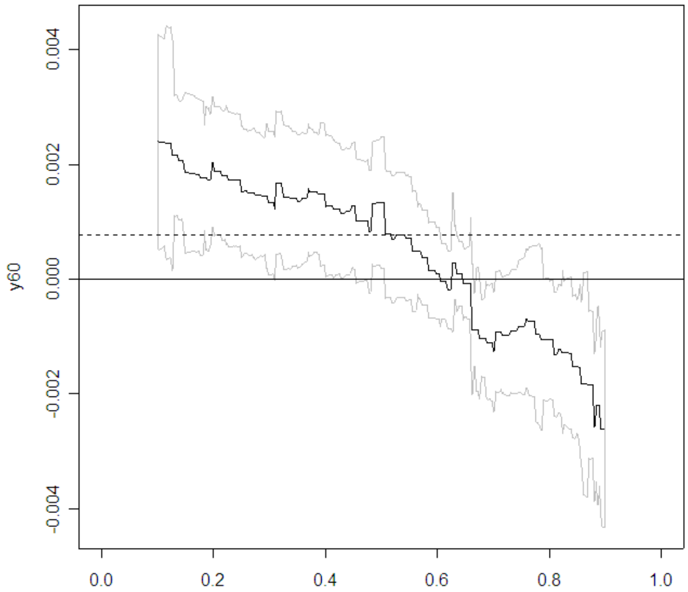

Figure 5 –Equation 2, Table 3
Barro Conditional

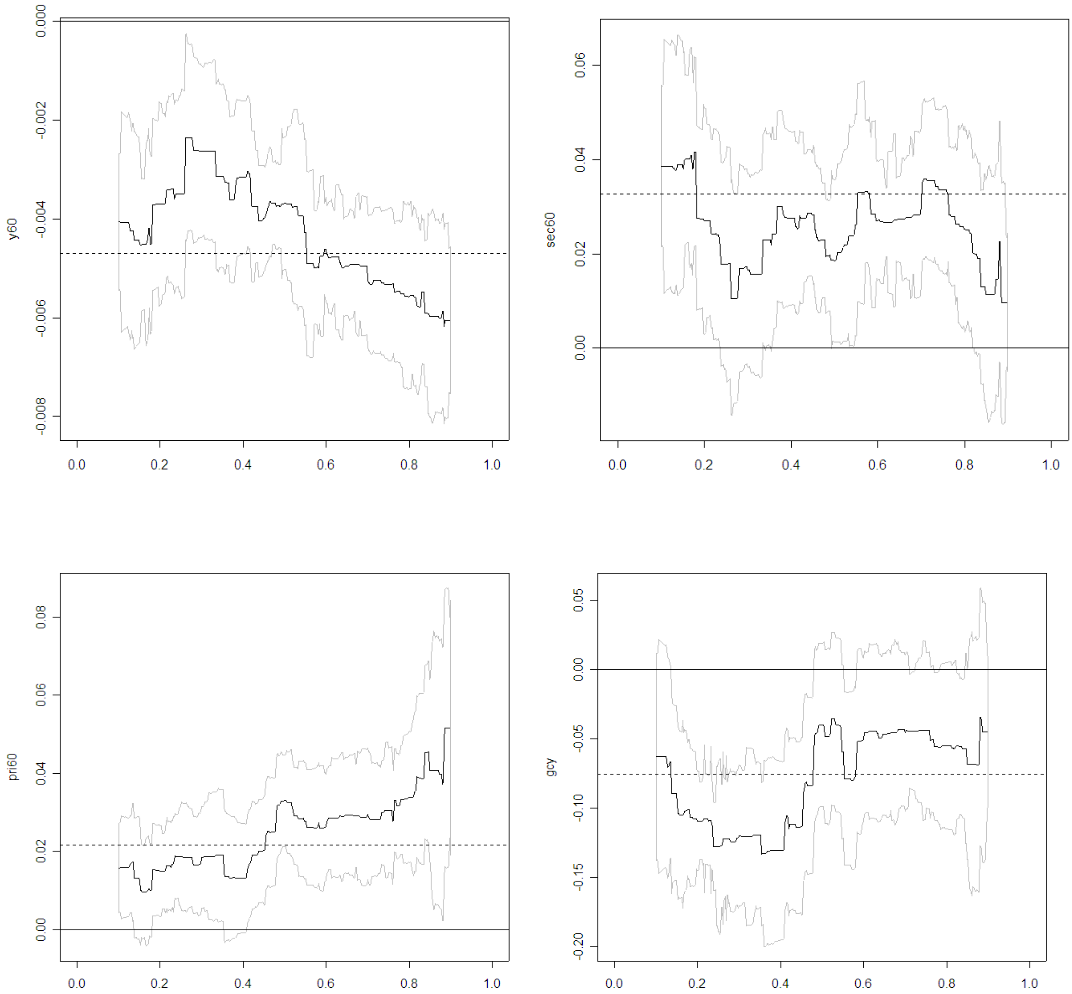

Figure 5 – Equation 2, Table 3
Barro Conditional, cont.

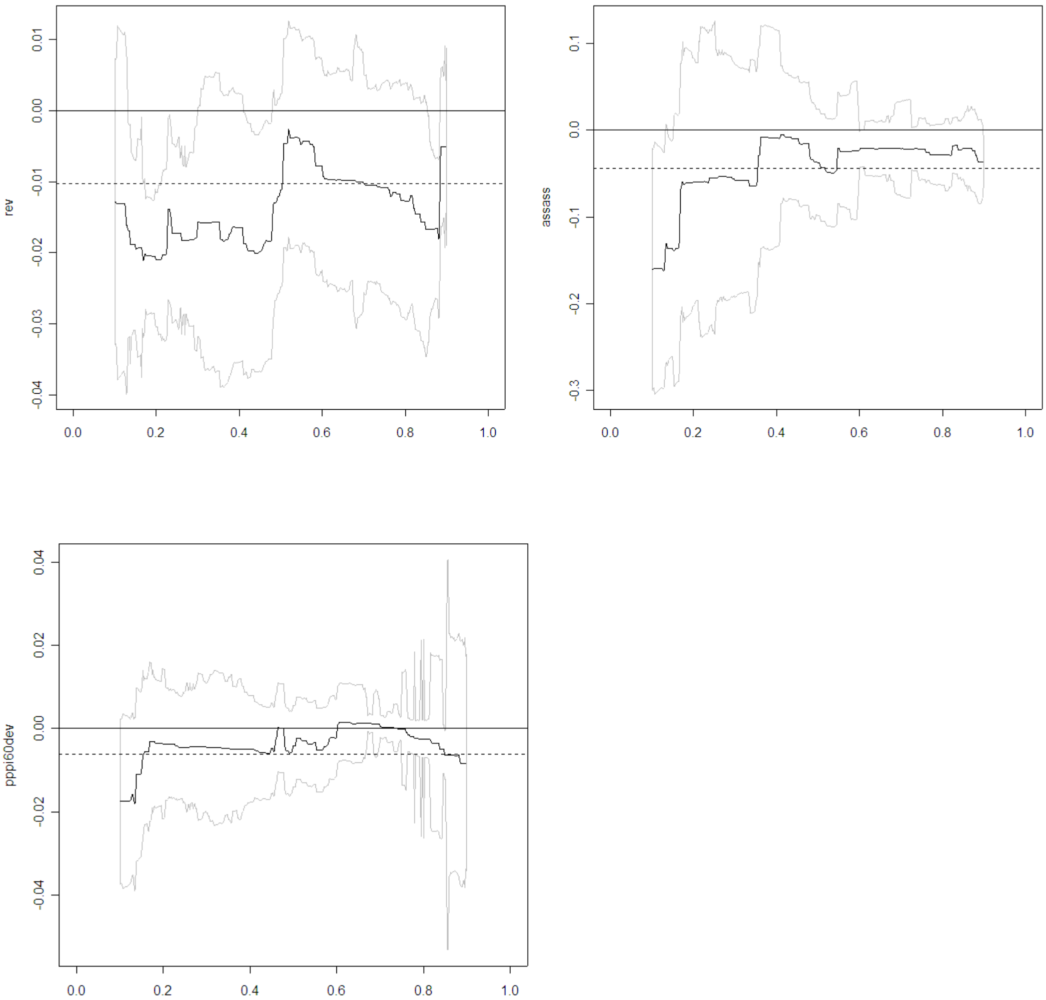

Figure 6 – Equation 3, Table 3
Barro Conditional

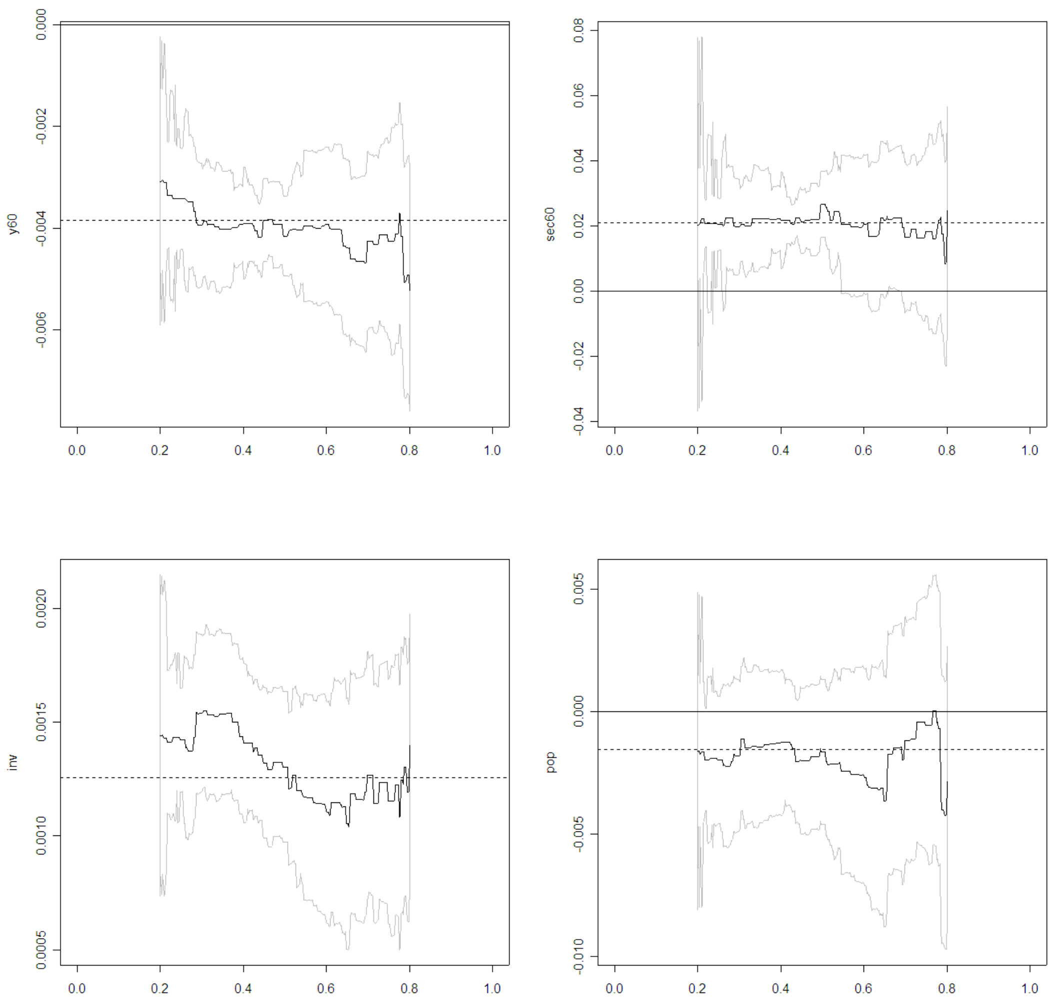

Figure 7 – Table 5, Equation 13.B
MRW Conditional, BG Data Set